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A CONSENSUS ACHIEVING PROCESS FOR GROUP PREFERENCE

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A CONSENSUS ACHIEVING PROCESS FOR GROUP PREFERENCE

by

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ABSTRACT

Often a group of individuals who must collectively make a decision differ in both their personal preferences for consequences to themselves and their social preferences for consequences to the entire group. We present a model for obtaining the weighting parameters of a surrogate utility function (SUF) for group decisions which is a weighted sum of personal utility functions. In our model, each individual makes interpersonal comparisons of the consequences to group members and iteratively revises such social preferences to account for the social preferences of others. The conditions under which this socialization process provides a unique SUF and the procedure for calculating the weighting parameters are described. Each stage of the socialization process excludes a portion of the Pareto frontier and any Pareto optimal alternatives that are excluded never cycle back into the set of alternatives being jointly considered at a later stage.

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A CONSENSUS ACHIEVING PROCESS FOR GROUP PREFERENCE

1. Introduction

Suppose a group of individuals must make decisions affecting the entire group when the individuals differ in both their *personal* preferences for consequences to themselves and their *social* preferences for consequences to the entire group. We present a model for obtaining a surrogate utility function (SUF) for group decisions which is a weighted sum of individual personal utility functions, the weights being determined by the social preferences of all individuals.

Many have written about the problem of constructing group preference functions (or social welfare functions). See Sen [9] for a good review of this literature. Most of this work has dealt with ordinal social welfare functions, although Keeney and Kirkwood [7], Keeney [8], Harsanyi [4], and others have considered the conditions leading to various mathematical forms for cardinal social welfare functions. One important form, the additive form (a weighted sum of individual personal utilities) is shown by Harsanyi [4] to be the only one satisfying bayesian rationality for both individuals and groups, and by Keeney [6] to satisfy some Arrow-like conditions on group choice.

Keeney and Kirkwood [7] suggest that the weighting parameters be set by a super decision maker or by mutual agreement of the group. Theil [10] shows how assuming a set of symmetry conditions results in a definition of the weighting parameters. Our model is distinct in that we use each individual's distinct social preferences to merge personal utility functions. There is no requirement that individuals exhibit the same decision behavior if acting for the group. Neither is there any assumption that the group

treats members identically.

In our model, each individual's social utility is revised to account for the social preferences of others. Under conditions identified below, this socialization process converges to a unique SUF. We can find such a SUF by solving a linear system of simultaneous equations.

This revision process has many desirable characteristics. Beginning with the Pareto optimal set of alternatives defined by personal utilities at iteration 0, each subsequent iteration has the potential of excluding a portion of the Pareto frontier. We show that alternatives which are excluded at any iteration never cycle back into the socially-Pareto optimal set defined by social utilities at any later iteration.

As an additional benefit, the model gives a measure of the extent of social concurrence between any pair of individuals. The matrix of social concurrence measures provides a complete alternative definition of the process from which we can construct the individual social utility functions and the SUF.

The model is intuitively appealing, explicit and computationally simple. While it is not a unique way to make group decisions, it does provide group consensus in situations where there may be no better alternative.

2. Preliminaries

Let u_i be the personal utility of individual i, i=1,2,...,n. This utility is based on the preferences of the individual for consequences affecting only himself, apart from the consequences to anyone else in the group. The level of utility is given by a utility function, $u_i(\underline{x}_i)$, where $\underline{x}_i$ is a vector of measurable attributes of the consequences to individual i. Individual i's social utility, u_i^1, is the desirability to i of the consequences

to the entire group. The level of social utility is given by a social utility function, $u_i^1(\underline{x})$, where $\underline{x}$ is a vector of attributes describing the consequences to the entire group, i.e. $\underline{x} = (\underline{x}_1, \underline{x}_2, \ldots, \underline{x}_n)$. Both $u_i(\underline{x}_i)$ and $u_i^1(\underline{x})$ are cardinal von Neumann-Morgenstern utility functions [11] obtained from individual i by direct preference assessment [1], [8].

It is natural for an individual's social utility to be a function of the personal utilities of the members of the group. The particular mathematical relationship between personal and social utilities is determined by the conditions on social preferences that are applied. Consider the following conditions paraphrased in our language from Keeney and Kirkwood [7]:

Condition 1. For any individual j, j=1,2,...,N, if the personal utilities of all the other N-1 members of the group are the same for all possible consequences, then the social utility of individual i, i= 1,2,...,n, over these consequences will be the personal utility of individual j.

Condition 2. If all the members of the group have the same personal utility function over the consequences, then the social utility function of individual i, i=1,2,...,n, should always yield the same decisions as the common personal utility function.

As shown by Keeney and Kirkwood [7], this leads to

Theorem 1. Given that conditions 1 and 2 hold, then the social utility function of individual i is given by

$$u_i^1(\underline{x}) = \sum_j^n w_{ij} u_j(\underline{x}_j). \qquad (1)$$

The two conditions seem plausible. The first one is a statement that if only one person cares about the personal consequences of some choice, individual i should be willing to let that person make the choice. The other

condition states that if every individual exhibits the same personal choice behavior, then individual i's social utility function should be consistent with this common personal choice behavior. These conditions only apply in extreme cases, when only one individual cares, or when all individuals feel the same way. But imposing the conditions restricts the way personal utility functions are aggregated to form social utility functions. Theorem 1 is a somewhat remarkable result, given the apparent weakness of the two conditions, but taken together, these two conditions have many subtleties (see [1], [7], [8]). Nonetheless, they are often justified and more often assumed for practical simplicity. Without claiming that they are always, or even usually, appropriate we adopt the additive relationship between personal and social utilities in our model, leaving more complicated realitionships to future work.

Whenever a relationship is postulated between personal and social utilities, an immediate difficulty arises concerning the way individuals make interpersonal comparisons of personal utility levels. This problem has received much discussion in the literature (see [9] for a review and references) with most writers arguing that personal utilities cannot be compared directly, since the definitions of zero utility and the unit of utility are arbitrary. However, some form of interpersonal comparison is the basis of any effort to construct social utility functions.

We avoid this dilemma by making interpersonal comparisons of consequences rather than interpersonal comparisons of utilities. The consequences are characterized by some set of physically measurable attributes, and all preference observations or questions are made in terms of these attributes. This notion of interpersonal comparison is

best illustrated by a simple example.

Suppose two individuals must decide how to set the thermostat in a room they both occupy. Their choices will have consequences measured by two attributes: the hourly cost of heating the room, C, and the temperature of the air in the room, T. We distinguish the cost and temperature consequences to individual one, (c_1,t_1), from the cost and temperature consequences to individual two, (c_2,t_2), to facilitate comparisons of consequences.

If we assume conditions one and two, then individual one's social utility function is given by

$$u_1^1(c_1,t_1,c_2,t_2) = w_{11}\, u_1(c_1,t_1) + w_{12}\, u_2(c_2,t_2) \tag{2}$$

This equation is completely specified if we know the way decisions are made in three situations:

1- when individual one makes decisions for himself when he is alone in the room. These decisions over various levels of c_1 and t_1 in some range give $u_1(c_1,t_1)$.

2- when individual two makes decisions for himself when he is alone in the room. These decisions over various levels of c_2 and t_2 in some range give $u_2(c_2,t_2)$.

3- when individual one makes decisions for both of them in the room. These decisions over various levels of c_1,t_1,c_2,t_2 in some range give $u_1^1(c_1,t_1,c_2,t_2)$.

We might actually observe the decision behavior or we might ask hypothetical questions to obtain the utility functions directly. The questions would be posed in terms of the attribute levels rather than utility levels, although in situation three, the respondent would be apprised of the utility function $u_2(\cdot,\cdot)$ obtained in situation two in order that his

consistency with the model of equation (2) might be checked.

The social preferences of individual one certainly depend on the identity of individual two. Thus if individual two is young, vigorous and relatively unaffected by temperature, the responses in situation three may well be different than if individual two is the aging grandfather of individual one. It is easy enough to make the interpersonal comparisons of consequences required in situation three; it is something we do routinely in social situations. We are only choosing to formalize the process and represent the choices mathematically.

We can calculate the parameters w_{11} and w_{12} in equation (2) once we know the decision making behavior in the three situations above. We must be careful in interpreting these parameters, however. They are not an importance weighting of individuals, i.e. if $w_{11}/w_{12} = 2$, we do not assume that individual one's personal preferences are twice as important as individual two's in determining the social preferences of individual one. Rather, the meaning of these parameters depends on the arbitrary scaling of the personal utility functions. Suppose that we scale all utility functions from zero to one over the same range, i.e. choose c^o,t^o and c^*,t^* and set

$$u_1^1(c^o,t^o,c^o,t^o) = u_1(c^o,t^o) = u_2(c^o,t^o) = 0$$

$$u_1^1(c^*,t^*,c^*,t^*) = u_1(c^*,t^*) = u_2(c^*,t^*) = 1.$$

Then the ratio w_{11}/w_{12} is the social importance to individual one of a change in the consequences to himself from (c^o,t^o) to (c^*,t^*) relative to a change in the consequences of individual two from (c^o,t^o) to (c^*,t^*).

Having related personal and social utilities for each individual is not enough to decide how to set the thermostat in the room. In general, we

expect the social utility function of the individuals to differ. The model described in the next section reconciles differing social utility functions in order to make collective choices.

3. The Basic Model

When there is no super decision maker and individuals do not agree on their social utility functions, group decision making is very difficult, yet decisions must be made. In these cases, we propose using a utility function for group decision making which is a surrogate of the social utility functions of the group members. Since we are assuming that all individual social utility functions have the additive form of equation (1), our SUF will also be a weighted sum of individual personal utilities. Our model for obtaining the weights is based on several assumptions.

Assumption 1. Each individual is willing to revise his own social preferences to account for the social preferences of others.

In forming social utility functions, individuals compromise personal preferences to account for the consequences to others. Assumption 1 is an extension of this notion of compromise to social utilities. As the individuals become aware of the social utilities of others, they account for this new information by revising their own social utility function, thereby moving up a step in their level of socialization and closer together in their social preferences. It is natural to view this socialization as a step-wise process where the social utilities at the next stage depend on the social utilities at the current time. As before, we assume the additive form of aggregation.

Assumption 2. The socialization process takes place in stages, with individual i's social utility at the rth stage of socialization given by

$$u_i^r = \sum_j^n v_{ij} \, u_j^{r-1}. \qquad (3)$$

If v_{ii} is very large in comparison to all other v_{ij}, then u_i^r is only a slight change from u_i^{r-1} under Assumption 2. Thus the v-parameters represent the weighting of the social utilities of others in the same way that the w-parameters represent the weighting of the personal utilities of others. To the extent that these two kinds of socialization are similar, a logical choice for the v-parameters would be to use the w-parameters determined from individual social utility functions.

Assumption 3. Each individual uses the same weights for revising his social preferences as for aggregating personal utilities, i.e., $w_{ij} = v_{ij}$, $i,j = 1,2,\ldots,n$.

We will use Assumption 3 to obtain the results that follow. However, we can easily develop similar convergence results for a more complicated rationale for selecting the v-parameters.

Let us define $\underline{u}$ and $\underline{u}^r$ column vectors with transposes $\underline{u}^T = [u_1,u_2,\ldots,u_n]$, $\underline{u}^{r^T} = [u_1^r,u_2^r,\ldots,u_n^r]$, and $\underline{\underline{W}}$ the matrix of elements w_{ij}. Then equations of the form of (3) can be written

$$\underline{u}^r = \underline{\underline{W}}\ \underline{u}^{r-1} = \underline{\underline{W}}^r\ \underline{u}. \tag{4}$$

4. Convergence

We now wish to determine under what conditions the social utility functions of the group members converge and how to compute the convergent weights of the resultant SUF. We may assume without loss of generality that

$$\textstyle\sum_{j=1}^{n} w_{ij} = 1 \tag{5}$$

This occurs naturally if u_i and u_i^r $(i=1,2,\ldots,n;\ r=1,2,\ldots)$ are scaled between zero and one over the same domain $[\underline{x}^o,\underline{x}^*]$. We note that $\underline{\underline{W}}$ is a stochastic matrix due to (5) and the observation that $w_{ij} \geq 0$ for all i and j. It has the properties of a one-step transition matrix of a first-order Markov process with n states. In the development which follows, we will exploit this

correspondence by applying limit theorems and computational methods from the theory of Markov chains.

The social utility functions of the individuals will converge if there is a weighting, $\underline{a} = [a_1, a_2, \ldots, a_n]$, such that

$$\lim_{r \to \infty} \underline{\underline{W}}^r_{ij} = a_j .$$

A common limit theorem, proved in virtually every book on Markov processes [5], is

Theorem 2. If the Markov process has no periodic states and if its recurrent states form a single closed communicating class (a single chain), then the process converges.

It is necessary to interpret the meaning of several terms in our consensus context.

In our process, individuals correspond to states. A communicating class consists of individuals who each incorporate the personal utility function of every other individual in the class into their social utility function. This may occur first hand when $w_{ij} > 0$ for all i and j in the communicating class or it may occur less directly through some higher order chain (i.e., $w_{ik} > 0$ and $w_{kj} > 0$ for all i and j in the communicating class).

Individual i is recurrent if at some stage of revision, u^r_j is dependent on individual i's personal utility function for all j.

Individual i is aperiodic, if there exists an R such that for all $r > R$, u^r_i depends on u_i.

When we do not have a single communicating class, we may at least segment the group into subsets of communicating classes, which can converge to a unique SUF.

Since each individual will normally assign a positive weight to his own personal utility, the aperiodicity requirement is trivially staisfied. Individual

i is recurrent in the case where every other individual does not exclude i's personal utility in their social utility.

An individual who assigns $w_{ii}=1$ is clearly not in a communicating class. He is analagous to the trap state of a Markov process. In this case, the process can only converge to u_i. To avoid this situation, we may wish to impose the conditions that all w_{ij} be positive, which additionally assures the recurrence of all individuals.

A transient state corresponds to an individual whose personal utility receives a weight of 0 in the social preferences of everyone else. Transient individuals do not preclude consensus, although the resulting SUF would not depend on the personal utility of the transient individual.

There are other convergence theorems with different and less stringent restrictions than those in Theorem 1. Another potentially useful ergodic theorem is

Theorem 3. If there exists a positive integer R such that at least one column of the matrix $\underline{\underline{W}}^R$ is positive, then a limiting state vector exists.

A column j of $\underline{\underline{W}}^R$ is positive if the social utility function of every individual at the Rth stage is dependent on the personal utility of j. In some cases this condition may be more easily demonstrated than the conditions of Theorem 1.

When convergence is assured we compute the limiting weights using another well known result from Markov process theory.

Theorem 4. If consensus is achieved the weights of the SUF

$$U = \sum_{i=1}^{n} a_i u_i \tag{6}$$

are given by

$$\underline{a} = \underline{a}\,\underline{\underline{W}} \quad \text{and} \quad \sum_{i=1}^{n} a_i = 1. \tag{7}$$

(Note: We must use equation (7b) because one of the n equations in (7a) is redundant.)

Consider two examples:

Example 1. Suppose

$$\underline{W} = \begin{bmatrix} 2/3 & 1/3 & 0 \\ 1/3 & 2/3 & 0 \\ 1/100 & 1/100 & 98/100 \end{bmatrix}$$

We have the equations,

a) $2/3\ a_1 + 1/3\ a_2 + 1/100\ a_3 = a_1$

b) $1/3\ a_1 + 2/3\ a_2 + 1/100\ a_3 = a_2$

c) $98/100\ a_3 = a_3$

d) $a_1 + a_2 + a_3 = 1$

Clearly c) is solved by $a_3 = 0$, with the other equations giving $a_1 = a_2 = 1/2$. Then $\underline{a} = [.5,.5,0]$ and the utility function of the third individual does not affect the surrogate utility function. We expected this to happen since individual three is transient.

Example 2. Suppose

$$\underline{\underline{W}} = \begin{bmatrix} 2/3 & 1/3 & 0 & 0 \\ 1/3 & 2/3 & 0 & 0 \\ 0 & 0 & 1/2 & 1/2 \\ 0 & 0 & 1/4 & 3/4 \end{bmatrix}$$

Individuals one and two communicate with each other and reach a consensus $1/2\ u_1 + 1/2\ u_2$ and individuals three and four communicate with each other and achieve a consensus $1/3\ u_3 + 2/3\ u_4$. However, the entire group does not achieve a consensus, since there are two communicating chains. At this point, even though we do not have complete consensus, we have at least obtained consensus within two interest groups.

The social utility functions at each stage of the socialization process have several desirable properties. It is well known that maximization of a weighted sum of individual personal utilities will give a Pareto optimal alternative. Hence, if any individual's social utility is maximized at any stage, a Pareto optimal alternative will result.

The Pareto optimality discussed above is based on personal expected utility. That is, an alternative is Pareto optimal if there is no other alternative having a greater expected personal utility for one individual without having a lower expected personal utility for some other individual.

We might consider a different Pareto optimality that is based on expected social utility. An alternative is socially-Pareto optimal if there is no other alternative having a greater expected social utility for one individual without having a lower expected social utility for some other individual. We would hope that the socially-Pareto optimal set is a subset of the Pareto optimal set. Indeed there is a socially-Pareto optimal set for every stage of the socialization process. We would hope that as we go through the socialization process, the socially Pareto optimal sets tend to become smaller and smaller. Indeed, we shall prove that, under weak conditions, the socially Pareto optimal set at stage r is a subset of the socially Pareto optimal set at stage r-1. This includes the case where the socially-Pareto optimal set at stage one is a subset of the Pareto optimal set (taken to be the socially-Pareto optimal set at stage zero).

Theorem 5. Let P_r be the Pareto optimal set for $\underline{u}^r$. If

a) $w_{ij} \geq 0$, $i,j = 1,2,\ldots,n$, and if

b) for $j = 1,2,\ldots,n$, $w_{ij} > 0$ for some i, then

$$P_r \subseteq P_{r-1}, \quad r = 1,2,\ldots$$

Proof: By definition, if an alternative $p \varepsilon P_r$, then there is no alternative having greater $\bar{u}_i^r$ for some i which does not have a lower $\bar{u}_j^r$ for some $j \neq i$.

Our proof is by contradiction. Suppose that $p \varepsilon P_r$ and $p \not\varepsilon P_{r-1}$. Then there must be some alternative q having the property

$$\bar{u}_i^{r-1}(q) \geq \bar{u}_i^{r-1}(p), \quad i = 1,2,\ldots,n \tag{8}$$

where at least one of the inequalities is strict. That is, there is some j with

$$\bar{u}_j^{r-1}(q) > \bar{u}_j^{r-1}(p). \tag{9}$$

Using condition a) and inequalities (8) and (9), we have

$$\begin{aligned} w_{gi}\, \bar{u}_i^{r-1}(q) &\geq w_{gi}\, \bar{u}_i^{r-1}(p) \\ w_{gj}\, \bar{u}_j^{r-1}(q) &> w_{gj}\, \bar{u}_j^{r-1}(p) \end{aligned} \qquad i,g = 1,2,\ldots,n$$

Using condition b), there exists some g such that $w_{gj} > 0$. Adding the above inequalities yields

$$\textstyle\sum_{i=1}^{n} w_{gi}\, \bar{u}_i^{r-1}(q) > \sum_{i=1}^{n} w_{gi}\, \bar{u}_i^{r-1}(p) \text{ for some } g.$$

Then

$$\bar{u}_g^r(q) > \bar{u}_g^r(p) \quad \text{for some } g.$$

Hence $p \not\varepsilon P_r$, a contradiction. Q.E.D.

By this theorem, each stage of the socialization process tends to shrink (at least not increase) the socially Pareto set of alternatives, provided a certain mild restriction on the matrix $\underline{W}$ is met. The restriction is essentially that there be no transient individuals. In most cases the restriction is easily met, although Example 1 is a case in which it is not.

Rather than converge to a unique SUF, we may wish to stop the socialization process after enough stages of revision to reduce the socially-Pareto optimal set to just a few alternatives. In this way we may greatly simplify the collective choice with a minimum of compromise. In some cases it may turn out that there

is just one alternative in the socially Pareto optimal set after only a few stages of the socialization process.

The two examples which follow illustrate what happens to the Pareto optimal set in the socialization process.

Example 3. In Figure 1 we have drawn a finite set of non-randomized alternatives and its Pareto optimal frontier (labelled A through H) in the space of expected personal utility for a group of two people. A particular social utility function

$$\bar{u}^r_i = \sum_{i=1}^{n} \underline{\underline{W}}^r_{ij}\ \bar{u}_j$$

is a vector in this space. Suppose

$$\underline{\underline{W}} = \begin{bmatrix} 1/2 & 1/2 \\ 1/4 & 3/4 \end{bmatrix} .$$

At iteration zero, the Pareto optimal set includes all alternatives on the northeast frontier. At iteration one, $\bar{u}^1_1 = 1/2\ \bar{u}_1 + 1/2\ \bar{u}_2$ and $\bar{u}^1_2 = 1/4\ \bar{u}_1 + 3/4\ \bar{u}_2$ and these vectors are indicated on Figure 2 along with their perpendiculars. An alternative is socially-Pareto optimal at this stage if, when we place the origin of these vectors at the alternative, there is no alternative in the shaded area. That is, there is no other alternative which is better in terms of the social expected utility of each individual. Hence at this point, only alternatives B and C are socially Pareto optimal.

Proceeding to the next iteration, $\bar{u}^2_1 = 3/8\ \bar{u}_1 + 5/8\ \bar{u}_2$ and $\bar{u}^2_2 = 5/16\ \bar{u}_1 + 11/16\ \bar{u}_2$. At this stage, then B is the only alternative in the socially Pareto optimal set.

At each stage, the vectors representing each individual's social utility move closer together than they were at the previous iteration, possibly removing part of the Pareto frontier in the process. In this example, the socially Pareto optimal set reduces rapidly, leaving only one alternative after the second

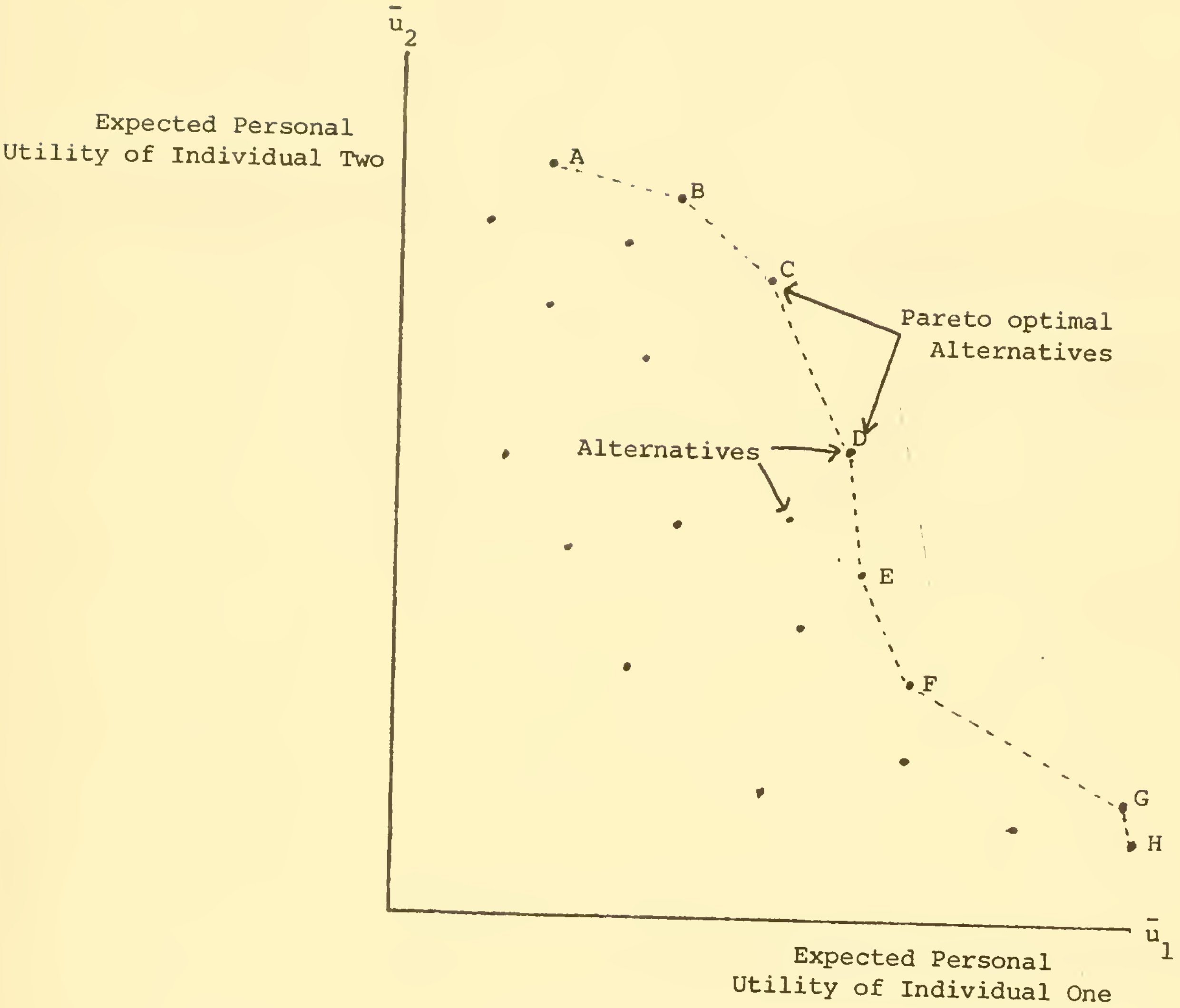
$\bar{u}_2$
Expected Personal Utility of Individual Two
A
B
C
D
E
F
G
H
Pareto optimal Alternatives
Alternatives
$\bar{u}_1$
Expected Personal Utility of Individual One

Figure 1. Pareto Optimal Boundary of an Alternative Set for Two Individuals

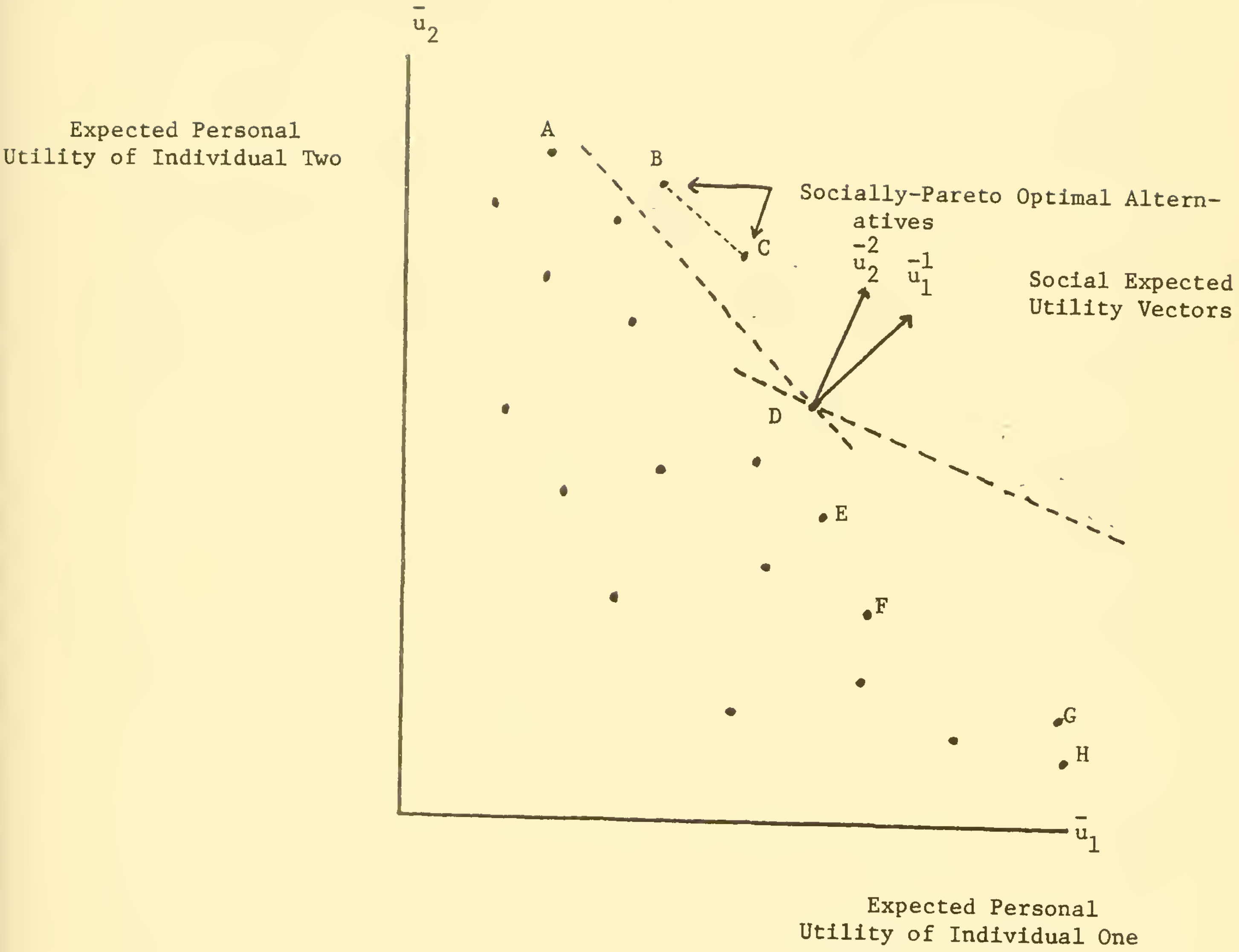
$\bar{u}_2$
Expected Personal Utility of Individual Two
A
B
C
D
E
F
G
H
Socially-Pareto Optimal Alternatives
$\bar{u}_2^2$
$\bar{u}_1^1$
Social Expected Utility Vectors
$\bar{u}_1$
Expected Personal Utility of Individual One

Figure 2. Social Expected Utility Vectors and Socially-Pareto Optimal Alternatives at Iferation One.

revision.

Example 4. Suppose there are three alternatives and three individuals with $\underline{\underline{W}}$ and the personal expected utility of the three alternatives given below:

$$\underline{\underline{W}} = \begin{bmatrix} .6 & .3 & .1 \\ .4 & .6 & 0 \\ .1 & .1 & .8 \end{bmatrix} \qquad \begin{array}{rccc} & \multicolumn{3}{c}{\text{Alternatives}} \\ & A & B & C \\ \bar{u}_1 = & .6 & .3 & .1 \\ \bar{u}_2 = & .3 & .6 & .1 \\ \bar{u}_3 = & .3 & .3 & .4 \end{array}$$

The initial Pareto optimal set P_o includes all three alternatives. On the second iteration, we have

$$\underline{\underline{W}}^2 = \begin{bmatrix} .49 & .37 & .14 \\ .48 & .48 & .04 \\ .18 & .17 & .65 \end{bmatrix} \qquad \begin{array}{rccc} \bar{u}_1^2 = & .447 & .411 & .142 \\ \bar{u}_2^2 = & .444 & .444 & .112 \\ \bar{u}_3^2 = & .354 & .351 & .295 \end{array}$$

Then P_2 includes only alternative A. We have converged upon a unique alternative while the individual social utility functions still exhibit some diversity, as indicated by $\underline{\underline{W}}^2$.

5. Misrepresentation, Collusion, and Concurrence

We have assumed a very cooperative model, rather than a strictly competitive model such as might be expected with game theory. A possible objection to our socialization process is that individuals may pervert the system by providing dishonest social preferences. We do not intend our model to be used in strictly competitive situations; in such cases a game model must be used. Nonetheless, there are many cases in which individuals would like to be fair and cooperative if they only knew a reasonable way to settle their differences.

The benevolence required in our model may arise because individuals are naturally altruistic, but more often an apparent benevolence will arise due to the retaliatory

affects of always demanding one's "own way." It might be useful to consider an analysis of the threat and counterthreat dynamics, but such an analysis would be difficult, even with many assumptions, and is somewhat tangential to our focus here.

We might avoid the problem of honesty to a certain degree by allowing individuals to make interpersonal comparisons only among others. In this case, each individual would act essentially as an arbiter in weighing the personal consequences of the complement set of individuals. He is motivated to be fair by the fact that those for whom he acts as arbiter will arbitrate over his consequences.

This approach will provide all of the off-diagonal elements of $\underline{\underline{W}}$, but the diagonal elements must also be set in some way. They might arbitrarily be made equal, or they may be taken from the average of the weights assigned to an individual by the other individuals, i.e.

$$w_{ii} = \sum_{\substack{j=1 \\ j\neq i}}^{n} w_{ji}/(n-1)$$

with w_{ij} normalized for each i such that $\sum_{j=1}^{n} w_{ij} = 1$. This obviously wouldn't work for n=2 and might be inadequate for small n.

Even with this modification some subgroup of individuals may agree to weight each other highly and everyone else insignificantly. The existence of such collusive subgroups is not necessarily undesirable. Collusive subgroups are cohesive, and the reason for our socialization process is to bring people together. It is no less desirable than having individuals working strictly on their own. The equity and minority opinion problems that are raised by collusion are inherent in virtually any process for group decision making. Processes similar to this one for more complicated

mathematical relationships between personal and social utilities than the additive form we use here will help solve some of the equity and minority opinion problems (see [1] for a discussion of the equity characteristics of more complex mathematical forms for a SUF).

One beneficial side effect of the mathematics of our process is a numerical measure of the choice concurrence of any pair of individuals. The first passage time, T_{ij}, of the Markov process is the expected number of transitions necessary to go from state i to state j. In our context, T_{ij} is a measure of the number of individuals between the ultimate social preferences of i (after an infinite number of revisions) and the personal preferences of j. That is, if individual 1 gives all of his weight to individual 2 and individual 2 gives all of his weight to the personal preferences of 3, then then $T_{13} = 2$. For lower values of T_{ij} less socialization is required for i's social preferences to concur with j's personal preferences. The concurrence index for the diagonals is given by

$$T_{ii} = 1/a_i \tag{10}$$

and for the off-diagonals, it may be found by solving simultaneous equations of the form

$$T_{ij} = 1 + \sum_{\substack{k=1 \\ k \neq j}}^{n} w_{ik} \, T_{kj}. \tag{11}$$

If for some reason the T_{ij}'s were known, we could also compute the matrix $\underline{\underline{W}}$ from them, as well as the vector $\underline{a}$ of convergent weightings when it exists.

Instead of assessing $\underline{\underline{W}}$, we might instead obtain the matrix $\underline{\underline{T}}$ by examining the distances between individuals in their socialization process. The alternate definitions of the model given by $\underline{\underline{W}}$ or $\underline{\underline{T}}$ may be used as

consistency checks on each other.

6. Discussion

We have presented a socialization process for achieving consensus on an additive SUF. The social utility functions at each stage of the process will select a Pareto optimal alternative if maximized and in the process of converging, parts of the Pareto optimal frontier become excluded from consideration, never to return to the set of alternatives to be jointly considered. The process is based on the interpersonal comparisons of consequences made by each individual in defining his social utility function, but it has an alternative interpretation in terms of the social concurrence of individuals.

The model is intuitively appealing and the computational scheme is explicit and simple. The assessment questions necessary to use it operationally are simple enough to be asked in a questionnaire or by an interactive computer program. The convergence requirements are well-defined and not overly restrictive. Even in cases where we cannot obtain or do not want convergence to a unique SUF, the model is valuable in identifying consensus of subgroups or in identifying socially-Pareto optimal sets of alternatives.

The critical assumption and the weakest link in the logic is Assumption 1, that individuals will revise their social preferences to reflect the social preferences of others. A study of the dynamic interactions of individuals when they know the way that their responses to assessment questions are to be used would be helpful to clarify the practical acceptability of these assumptions.

There are some other disadvantages of the model, including the time required to use it, the accuracy of the weighting parameters, and the problem of determining when it should be used. The very explicitness of the model might be considered a disadvantage, particularly for highly

politicized situations where individuals would prefer to have a process that cannot be easily reviewed and criticed. It is important not to ignore nor minimize these difficulties, but it should be recognized that they are inherent in any formal approach to group decision making. Nonetheless, when group decisions have to be made, the formal model we propose will often provide a means of quickly coming to a suitable agreement, but one which is based on known assumptions rather than obtained in an *ad hoc* manner.

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